FLOW CONTINUITY OVER A RIEGEL, STORGLACIAREN, SWEDEN

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INTRODUCTION

In its ablation zone, Storglaciären flows over several riegels, or transverse bedrock ridges. This report describes spatial patterns of measured surface velocity just upglacier of one riegel during the winter of 2000-2001. The data indicate that as the glacier approaches the constriction in its valley, which is created by the lower riegel, the surface velocity increases. Such extension is not uncommon but is contrary to the precept that a glacier's ablation zone is characterized by compression. There are two ways to explain this velocity "anomaly". The first considers the glacier to be a beam bending as it flows over the riegel, causing stretching of the upper surface and acceleration there. The other explanation assumes the glacier to be an incompressible fluid flowing through an irregular channel. Flow continuity would require that the glacier velocity be higher through reaches of the channel that are smaller in area. Thus, the glacier would accelerate over the riegel. This latter argument is explored in the context of the measurements from Storglaciären and is shown to be a sufficient explanation.

BACKGROUND

The principle of flow continuity emerges from conservation of matter and is fundamental to the flow of all kinds of fluids through space. For an incompressible flow, continuity requires:

$$\int_x \boldsymbol{r}\vec{\mathbf{v}} \bullet dA \tag{1}$$

where x is a distance parallel to flow, $\boldsymbol{r}$ is fluid density, $\vec{\mathbf{v}}$ is the velocity in the x-direction, and A is the area of the plane perpendicular to x through which the flow takes place (Fanger, 1970). As applied to fluid flow through a channel whose cross-sectional area varies in the x-direction, this simply means that the product of flow velocity and area at one point must be equal to the product of the flow velocity and area at another point. Consequently, for example, a fluid flowing from a pipe of large diameter must accelerate when it enters one of smaller diameter. Glacier ice is a fluid, which with some strings attached, must comply with continuity.

LIMITATIONS AND ASSUMPTIONS

The temporal and spatial scales of the measurements discussed in this report are limited, and a more exhaustive measurement program would arguably yield results that are somewhat different from those presented here.

In the vicinity of bed irregularities, glacier flow can be complex, and the surface velocity patterns are not necessarily entirely correlated with the velocity patterns at depth in the glacier (e.g., Hooke *et al.*, 1987). Such flow complexity could produce uncertainties in the calculation of mean velocity using only surface data.

Finally, there are many mass inputs and sinks along a valley glacier's length. Therefore, a rigorous equation for continuity of glacier flow must include additional terms which account for gains and losses of ice mass. In order to simplify this analysis, most inputs and variables have been combined or neglected.

DATA

Coordinates of 24 stakes were measured during September of 2000 and April of 2001 with a Javad differential GPS unit. The coordinates were obtained with a roving unit which was referenced to a base station located at the "Enqvist rock" control point near the Tarfala Field Station. Data points were retrieved in WGS84 latitude-longitude format and entered into ArcView, where distances were computed in meters. These distances were taken to represent the displacement of the ice surface during the winter. Average winter velocities calculated from the displacements are plotted below.

SURFACE VELOCITY FIELD (m/a, winter 2000-2001)

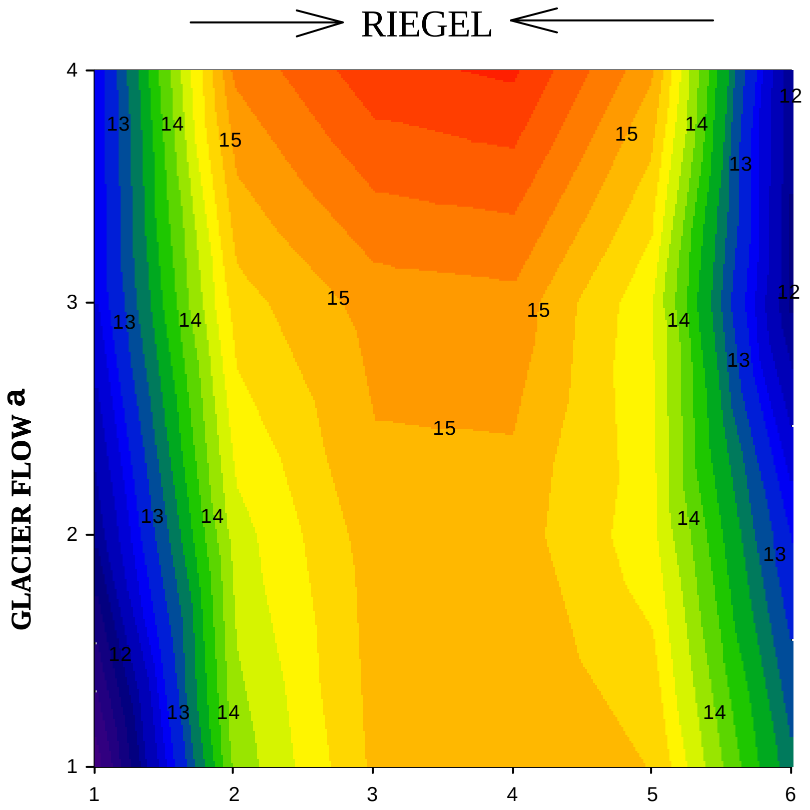

FIGURE 1: Colored contour graph of surface velocity field as measured by GPS.

ANALYSIS

A simple flow continuity equation for an idealized glacier can be written:

$$h\left(\frac{\partial \bar{u}}{\partial x}\right) = b - \bar{u}\left(\frac{\partial h}{\partial x}\right) - h\left(\frac{\partial \bar{\boldsymbol{u}}}{\partial y}\right) \qquad (2)$$

where h is ice thickness, $\partial \bar{u} / \partial x$ is the change in average downglacier velocity along the glacier flowline, b is the mass input or output per unit time resulting from accumulation or ablation, $\partial \bar{\boldsymbol{u}} / \partial y$ can be viewed as the change in average velocity perpendicular to the flowline that results from changes in valley width (convergence or divergence), and $\partial h / \partial x$ is the longitudinal change in ice thickness (modified from Paterson, 1994, p. 257). In the reach of Storglaciären that was surveyed for this report, $\partial \bar{\boldsymbol{u}} / \partial y$ becomes slightly negative (narrowing), $\partial h / \partial x$ decreases (thinning), and $\partial \bar{u} / \partial x$ increases (extension), as shown in Figure 1.

To apply this equation ,we must first find the average velocities at two index points. Assuming that longitudinal stress gradients are small, the depth-averaged ice velocity is:

$$\bar{u} = u_s - \frac{2}{(n+1)(n+2)} \left(\frac{S_f \, \boldsymbol{r} g \sin \boldsymbol{a}}{B} \right)^n H^{n+1} \qquad (3)$$

where u_s is the measured surface velocity, *n* is the flow law exponent for ice, S_f is the valley shape factor, *g* is the acceleration due to gravity, $\boldsymbol{a}$ is the slope of the glacier surface, *B* is the flow-law parameter, and *H* is the ice thickness (Hooke, 1998). Because the surface velocity measurements were close together, all variables except for the velocity and ice thickness are taken to be constant and are listed in Appendix 1. Calculating the mean flow velocity with depth along a flowline upglacier from the riegel and in the vicinity of the riegel yields mean velocities of 14.5 m/a and 15.7 m/a, respectively. Incidentally, this indicates an increase in velocity of 1.2 m/a over a flowline distance of just under 296 meters, which would yield a surface strain rate of roughly 0.06 a^{-1}, possibly significant enough to form crevasses (cf. Paterson, 1994, p. 188). Entering these mean velocities and the pertinent geometrical parameters into the continuity equation (parameter values are discussed in Appendix 1), we find that the left-hand side of the equation equals 0.69 m/a and the right-hand side comes to *b* + 0.705 m/a. This leaves the value of *b* at -0.015 m/a, which must represent all mass flux lost through ablation and other processes along this reach. Considering the relative proximity of this study reach to the equilibrium line where *b* = 0, this result seems reasonable.

CONCLUSIONS

The result of this brief analysis suggests that the increase in measured surface velocity can adequately be explained by simple continuity of glacier flow through a constriction of the valley. However, this conclusion is uncertain due to the fact that longitudinal stress gradients were assumed to be negligible in the calculation of mean velocity. On the contrary, Jansson (1997) suggests that “pulling” from downglacier or “pushing” from upglacier is likely to be the mechanical process that is responsible for driving the ice over the riegel.

APPENDIX 1

Parameter values used for calculation of mean velocity [eqn. 3]:

$n = 3$
$S_f = 0.5$
$\boldsymbol{r} = 900$ kg/m^3
$g = 9.81$ m/s^2
$\boldsymbol{a} = 0.068$
$B = 0.19$ Mpa a

Parameter values used in the continuity analysis [eqn. 2]:

$h = 170$ m
$\hat{u} = 14.55$ m/a
$\frac{\partial \bar{u}}{\partial x} = -0.00405$ m/a/m
$\frac{\partial h}{\partial x} = -0.169$ m/m
$\frac{\partial \bar{\boldsymbol{u}}}{\partial y}$ = change in width of valley/time taken to span reach/total width of valley = -0.01 m/a/m

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